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A Model of Endogenous Quality Management

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A Model of Endogenous Quality Management

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Abstract

This paper is concerned with product quality, defined as a kind of durability. Existing models of product quality (in the sense considered here) depend on the idea of signalling, itself driven by an informational asymmetry dictated by Nature."The paper proposes an alternative approach, which endogenises the quality management process. A model is developed which is applicable to the markets for consumer durables and for some intermediate goods. Both competitive and monopolistic markets are considered, and some comparative static results are obtained.

1. INTRODUCTION

In the management literature "product quality" is defined extremely broadly. The term has been used to refer to safety, availability, maintainability, reliability, usability and even price (see e.g. Besterfield, 1986). Generally speaking quality is best thought of as a characteristic of the product with the property that all consumers prefer more of it to less, **at a given price**. Some such characteristics will be known to the consumer before purchase, while others will not. For personal computers, for example, the former type of characteristic might include the size of the processor chip (286, 386, 486, Pentium etc.) or its speed (166Mhz., 200Mhz., 450Mhz. etc.), while the latter type might include product lifetime, repair costs etc.. Most goods have characteristics of both types, though the second notion of quality raises more interesting questions for firms, consumers, regulators and for economic theory. It is the notion of quality dealt with in this paper.

Firms devote considerable resources to influencing the quality of their products. This influence operates at the level of product design, production process and post-production quality control. The unity of the quality management process is often stressed in the management literature. The distinction between production and quality control decisions is frequently blurred. For example, a firm may seek to raise its production quality by, in effect, demanding tighter quality control from its components suppliers. Thus a production decision in one firm is inseparable from a post-production quality control decision in another. The model developed in this paper incorporates production, quality control, warranty and pricing decisions into the firm's overall (expected) profit maximising behaviour. Product design decisions are not considered.

In the model presented here, both firms and consumers will be assumed to be ignorant at the moment of purchase, as to the quality of any given product, though they will be assumed to know the probability distribution of quality. Thus the model is one of imperfect but symmetric information. It will further be assumed that the firm, though just as ignorant as consumers, is

less risk-averse. There thus arises a demand, on the part of consumers, for insurance. This might, for example, be provided in the form of a product warranty offered by the firm, or an insurance policy provided jointly with the product. In the case of intermediate goods "consumers" may be thought of as firms and "warranties" as compensation clauses built into standard supply contracts. Heal (1977) develops a model, involving warranties, which adopts precisely these informational assumptions. He remarks:

"Typically the quality control is sufficiently imperfect that no one (i.e. neither seller nor buyer) will know in advance of (a product's) use what (its) quality will be, and consequently some form of guarantee will be offered." (Remarks in brackets added)

In Heal's model the firm is assumed to produce a probability distribution of qualities which is simply taken as given. He does not seek to model the process by which the firm attempts to alter that distribution. In this paper this process is modelled. In particular the firm is able to influence the distribution of quality in its marketed output, both by production (choice of technique) decisions and by quality control decisions. It will also be able to offer a product warranty to the market. A standard problem, often assumed away, in the literature on quality, is that of moral hazard on the part of consumers. If consumers can themselves influence the probability or size of a claim under the warranty, for example by failing to take proper care of the good during consumption, then the economic role of warranties may be reduced. See, for example McKean (1970), Oi (1973) and Priest (1981). For simplicity moral hazard will be assumed away in this paper. Warranties, whether voluntary or legally compelled, have an important bearing on quality management decisions because the higher the quality of a firm's marketed output, the lower the likely warranty costs experienced by the firm. Thus warranties provide the firm with an incentive to market high quality products. This connection between warranties and quality management has been apparent to managers for some time. Wright (1980), for example, describes events at General Motors:

"I instituted a programme for testing and repairing faulty cars as they came off the assembly line - and the results were phenomenal. It cost about \$8 a car, which drove The Fourteenth Floor up the wall. But I figured one way or the other we would end up fixing the defects or paying to have them fixed through recall campaigns or dealer warranty bills..... The internal quality control audit revealed a 66% improvement in the quality of a Chevrolet coming off the assembly line between 1969 and 1973 models. And most important, warranty costs of our new cars were down substantially."

The existing economics literature deals with both the notions of quality discussed above. When quality is known to consumers before purchase, the focus of interest is screening. The seller will be ignorant as to the preferences of any individual consumer though he may be assumed to know the distribution of preferences across the population. His problem then is to provide a price-quality schedule, perhaps along with a warranty arrangement, to the market with a view to screening consumers and thus extracting the maximum surplus from them. The firm produces a "product line", deliberately differentiating his product by quality. An obvious example is personal computers: most manufacturers produce a product line involving different processing speeds, amount of RAM, size of hard disc etc. Authors who develop screening models of quality include Mussa and Rosen (1978) and Matthews and Moore (1987). Signalling models, by contrast deal with a different asymmetry of information, namely one concerning the product itself. Such models are driven by exogenous "type uncertainty". That is to say "Nature" dictates a firm's quality which is then known to that firm but not to consumers. The firm's problem then is to signal its quality to consumers using price, warranties and possibly advertising. In a repeat purchase framework the firm may be able to build up a "reputation" for quality. Authors who develop signalling models of quality include Grossman (1981), Milgrom and Roberts (1982, 1986), Kreps and Wilson (1982), Klein and Leffler (1981), Shapiro (1983) and McClure and Spector (1991).

This paper is concerned with product quality that is unknown to the consumer before purchase. Screening models do not deal with this notion of quality. Signalling models, by contrast, do

analyse this concept of quality, but they do so in a way which treats quality as exogenous and not affected by the firm's decisions. Both screening and signalling models place the emphasis of the analysis on an **asymmetry** of information. The approach of this paper is to allow firms' choice of technique and quality control decisions to influence product quality under conditions of imperfect but symmetric information.

Section 2 describes a theoretical approach to modelling quality management and section 3 develops this approach into a tractable model. Section 4 describes the demand side of the economy, while section 5 covers the supply side. Equilibrium and comparative statics are discussed in section 6, and section 7 concludes.

2. AN APPROACH TO MODELLING QUALITY MANAGEMENT.

This paper will distinguish two main classes of decision which influence product quality, namely choice of technique decisions and quality control decisions. "Quality" will be modelled as a variable (q) drawn from the interval $[0,1]$ and the firm's produced output will be modelled as a frequency distribution function $X:[0,1] \rightarrow R_+$ (see figure 1). The firm will be assumed able to expand or contract the scale of production by a factor $l \geq 0$. The firm will be assumed to have available a spectrum of techniques, indexed by a real variable t , each generating a different distribution function. We may therefore index the distribution function X with the variable t . That is to say the volume of production with quality lying between a and b is given by:

$$l \int_a^b X_t(q) dq \quad (1)$$

By choosing technique t , the firm can influence various parameters of the distribution function X , such as the mean and variance. Thus the firm cannot determine the quality of each individual product but it can determine the average quality of its production. By varying l the firm can also determine the volume of production.

In addition to choice of technique decisions it will be assumed that the firm can undertake some post-production quality control. By applying techniques such as product testing and batch sampling, it can obtain some information as to the quality of its products. It can then decide which products to market and which to scrap or rework. For example it may set a quality threshold and market all products believed to be above the threshold and scrap or rework all products below it. Such a process, which will be termed a "quality filter", need not be perfectly accurate, so that low quality products may inadvertently get marketed while high quality ones are unnecessarily reworked or scrapped. Formally a quality filter may be defined as a function $P(q)$ such that $P:[0,1] \rightarrow [0,1]$ (see figure 2), where $P(q)$ is the probability that a product of quality q will be marketed. Thus the total marketed output of the firm will be given by:

$$\int_0^1 P(q) X_i(q) dq \quad (2)$$

Quality control decisions thus amount to choosing parameters of the function P . In taking such decisions the firm will clearly take into account the scrap or rework value of products which fail the quality filter. For products such as silicon chips with virtually costless disposal this scrap/rework value may be taken to be zero. For products requiring costly disposal, the rework value may be taken to be negative, while for goods such as domestic appliances, which can be readily reworked, it will be positive.

(figures 1 and 2 near here)

In general, quality management decisions will be taken in the light of demand conditions, production costs, expected warranty costs, rework value, and competitive pressures. While the approach described above does capture the essentials of quality management, it is too general to be readily applied. In the next section a less general, but more tractable version of this approach is developed.

3. A TRACTABLE MODEL OF QUALITY MANAGEMENT

The notion of "quality" which this paper will focus on is "durability", defined as the probability that the product will not break down during a particular period (the warranty period). Some products, such as light bulbs or silicon chips either function or fail to function, so that this approach presents no difficulties. For other products, such as motor vehicles, which are subject to different degrees of breakdown, it is obviously a simplification. For simplicity the warranty period will be treated as exogenous throughout the paper. The firm will be assumed to produce a range of physical types of its product. However the product's lifetime will not be assumed to be uniquely determined by its physical type: there will be other, random, factors at work. In the case of light bulbs or silicon chips, these might include air temperature or mains voltage fluctuations for example. Thus, for each physical type of product there will be a probability that it will not break down during any given time interval (such as the warranty period). We may therefore identify "type" with "probability of not breaking down during the warranty period", that is with "quality". It will be assumed later that, even though consumers do not know the quality of any particular product before consumption takes place, they do know the probability distribution of qualities. The same information will be assumed to be available to the firm, and the situation is thus one of imperfect but symmetric information.

The firm's distribution of produced qualities (the X function of section 2) will be taken to be uniform with mean b and a fixed variance. The firm will be assumed to have available a spectrum of techniques each corresponding to a different mean quality of produced output (i.e. a different b): higher quality techniques will be assumed more costly to operate than lower quality ones. A distribution function of produced qualities is depicted in figure 3. Note that it is uniform on the interval $[b-h, b+h]$.

(figure 3 near here)

The firm can thus influence the quality of its products by choice of technique. It will also be assumed to undertake some post-production quality control. In particular the quality filter of section 2 (i.e. the P function) will be taken to be a step function. That is the firm can set a threshold or "stringency" level (a) of quality such that all products exceeding that level are marketed, while all products falling short of it are reworked or scrapped. Such a quality filter is depicted in figure 4. Note that it is a perfectly accurate quality filter in the sense that there is a zero probability of "bad" products being marketed or "good" ones being reworked or scrapped. Scrap/rework value will be taken to be constant at v per unit. Figure 5 depicts the effect of combining production and quality control. Note that $b - h < a < b + h$.

(figures 4 and 5 near here)

4. THE DEMAND SIDE

It will be assumed that there are D risk-averse consumers with identical preferences, each consuming $1/D$ units of the good. (Thus the total marketed output is equal to unity.) In the case of an intermediate good, these consumers can be thought of as other firms. Buyers know nothing about the quality of the particular goods they are buying, though they can deduce the distribution of qualities across marketed output. The seller offers a contract to the market which consists of a price (p) and a warranty payment (β). This contract requires the seller to make a payment of β per unit, to the buyer if the product breaks down within the warranty period. There will be assumed to be no costs of enforcing such a contract. The warranty contract may be thought of a consumer guarantee or an insurance policy provided jointly with the product or, in the case of an intermediate good, a compensation clause built into the standard inter-firm supply contract. The problem of moral hazard (on the part of buyers) is assumed away.

Each consumer is assumed to have a budget of M and to receive a stream of services from the product. If the product does not break down within the warranty period, this stream of services, derived from $1/D$ units of product, is worth R to the consumer, while if it does break down the stream of services is worth αR ($\alpha < 1$). Thus the consumer receives income M if she doesn't buy the product, $x = M - p/D + R$ if she buys and the product doesn't break down, and $y = M - p/D + \alpha R + \beta/D$ if she buys and the product does break down. She is assumed to choose a contract so as to maximise expected utility $V(x,y) = \Pi U(x) + (1-\Pi)U(y)$ where Π is her subjective probability that the product will not break down during the warranty period. Sellers' maximising decisions are clearly subject to the **voluntary participation constraint**:

$$V(x,y) \geq U(M). \quad (3)$$

If this constraint were not satisfied the consumer would clearly not consume the product at all. Note that the consumer is assumed risk-averse, so that $U''(.)$ must be strictly negative. Throughout the paper it will be assumed that the consumers' subjective probability that the product does not break down is equal to the objective probability (i.e. the average quality of marketed output, Q). This average quality is determined by the quality management decisions of sellers, and of course the consumer cannot observe these decisions directly. Moreover the model set up here does not admit repeat purchasing, so the consumer cannot learn about average quality over time. Nonetheless, there is no asymmetry of information so the consumer could in principle deduce the seller's quality control decisions and thus deduce average quality.

The market demand curve always has the step function appearance of figure 6, but the price p^* at which the step occurs will depend on the warranty payment (β) and the average quality of marketed output (Q), which, in turn, depends on the quality management decisions of firms.

(figure 6 near here)

5. THE SUPPLY SIDE

Quality costs are discussed at some length in the management literature. Groocock (1986) points out :

"Because the products might be defective, they must be inspected and tested. This results in appraisal costs.....Products may also fail a test or inspection, or may fail in the hands of customers. Failure costs are then incurred..(since the firm) must rework or replace the failed product during manufacturing, or replace or repair the product for customers, for example, under warranty." (Groocock, 1986, p53)

The model developed here formalises these costs as quality control costs, rework costs and expected warranty costs. Sellers will be assumed to be risk-neutral maximisers of expected profit. Average and marginal production costs, for a given technique, will be assumed constant. Thus the volume of marketed output will be determined by the size of the market (and will thus be normalised to unity, given the assumptions of Section 4 above) and the focus of attention is thus placed on the firm's quality management decisions. The firm's expected profit is given by:

$$\Phi = \text{revenue} - \text{production costs} - \text{quality control costs} + \text{rework value} - \text{expected warranty costs.} \quad (4)$$

Note that rework value might be positive, zero or negative. From figure 3. it is clear that marketed output (z) is given by:

$$z = (b + h - a)K \quad (5)$$

By the assumptions of section 4 above, z is normalised to unity. Thus we have:

$$K = 1 / (b + h - a) \quad (6)$$

It is now easy to derive an expression for N , the firm's total produced output.. From figure 5:

$$N = 2hK = 2h / (b + h - a) = N(a, b) \quad (7)$$

It is clear from figure 5 that the average quality of marketed output is given by:

$$Q = (b + h + a) / 2 = Q(a, b) \quad (8)$$

Average and marginal production costs will be assumed constant for a given technique, but increasing in the quality level (b) of that technique. This is captured by the production cost function:

$$\text{production costs} = \gamma b N \quad (\gamma \text{ a constant}) \quad (9)$$

Quality control costs will be assumed to depend only on the volume of products processed and not on the stringency level chosen. In many quality control processes (such as batch sampling) the entire output is not processed, but is reasonable to assume that the quantity processed is proportional to the quantity produced. It will further be assumed that there are no economies or diseconomies of scale in quality control. A suitable quality control cost function is therefore:

$$\text{quality control costs} = \delta N \quad (\delta \text{ a constant}) \quad (10)$$

Since a total of one unit is marketed, the volume of reworked (or scrapped) output is $N - 1$.

Since each reworked product is worth v to the firm, we have

$$\text{total rework value} = v(N - 1) \quad (11)$$

The expected number of products failing during the warranty period is simply $1 - Q$. Thus we have:

$$\text{expected warranty costs} = \beta(1 - Q) \quad (12)$$

Combining equations (9) to (12) and substituting in equation (4) we obtain an expression for the firm's expected profit:

$$\Phi = p - \gamma bN(a,b) - \delta N(a,b) + v(N(a,b) - 1) - \beta(1 - Q(a,b)) \quad (13)$$

6. EQUILIBRIUM AND COMPARATIVE STATICS

6.1 Definitions of Equilibrium

Two types of equilibrium are considered. A monopoly equilibrium is a vector (p, β, a, b) which maximises expected profit (Φ) subject to the voluntary participation constraint (equation (3)). This constraint binds in monopoly equilibrium. A competitive equilibrium is a vector (p, β, a, b) which maximises expected utility (V) subject to the constraint $\Phi \geq 0$. This constraint binds in competitive equilibrium. That is to say a zero expected profit condition holds. This may be interpreted as a consequence of free entry or of Bertrand competition.

6.2 Monopoly Equilibrium

To characterise a monopoly equilibrium we take a multiplier λ for the voluntary participation constraint (equation (3)) and form the Lagrangian:

$$L(p, b, a, b) = \Phi + \lambda [V(x, y) - U(M)] \quad (14)$$

Substituting for Φ from equation (13), we obtain:

$$L(p, \beta, a, b) = p - \gamma bN(a,b) - \delta N(a,b) + v(N(a,b) - 1) - \beta(1 - Q(a,b)) + \lambda[V(x,y) - U(M)] \quad \dots(15)$$

Note that:

$$x = M - p/D + R \quad (16)$$

and:

$$y = M - p/D + \alpha R + \beta/D \quad (17)$$

On the assumptions of section 4 we have $\Pi = Q$ and thus:

$$V(x,y) = Q(a,b)U(x) + (1 - Q(a,b))U(y) \quad (18)$$

It is now easy to establish:

Proposition 1. The voluntary participation constraint binds in monopoly equilibrium

Proof. Differentiating (15) w.r.t. p we obtain one of the first order conditions for an interior maximum:

$$L_p = 1 + (\lambda / D)[-QU'(x) - (1 - Q)U'(y)] = 0 \quad (19)$$

Hence $\lambda \neq 0$ and, by complementary slackness, the voluntary participation constraint must bind.

It is now straightforward to prove:

Proposition 2. In monopoly equilibrium the warranty payment (β) is equal to $D(1 - \alpha)R$, and thus the consumer receives the same income stream regardless of whether the product breaks down or not, and is thus fully insured (i.e. $x = y$).

Proof. Differentiating (15) w.r.t. β yields another first order condition for an interior maximum:

$$L_\beta = (Q - 1) + (\lambda / D)(1 - Q)U'(y) = 0 \quad (20)$$

$$\Rightarrow (\lambda / D)U'(y) = 1 \quad (21)$$

Using (21) to substitute for λ/D in (19) we obtain:

$$\begin{aligned} 1 - [QU'(x) / U'(y) + 1 - Q] &= 0 \\ \Rightarrow U'(x) &= U'(y) \end{aligned} \quad (22)$$

Since $U''(\cdot)$ is strictly negative, the function $U'(\cdot)$ is invertible. It follows from (22) that $x = y$ and hence (by (16) and (17)) that $\beta = D(1 - \alpha)R$, as required.

Combining Propositions 1 and 2, it is straightforward to establish:

Proposition 3. The monopoly price is given by $p = DR$. Thus the monopolist extracts all the surplus.

Proof. Since the voluntary participation constraint binds (Proposition 1) we have:

$$QU(x) + (1 - Q)U(y) = U(M)$$

From Proposition 2 we have $x = y$ and hence: $U(x) = U(M)$. The function $U(\cdot)$ is invertible because $U'(\cdot)$ is strictly positive. Hence $x = M$ and the result follows, using (16) and (17).

Attention is now turned to equilibrium quality management decisions. We first derive expressions for b (the production quality level) and a (the quality control stringency level).

Proposition 4. In monopoly equilibrium the production quality level (b) is given by:

$$b = \gamma h / \beta + (v - \delta) / \gamma \quad (23)$$

In monopoly equilibrium the quality control stringency level (a) is given by:

$$a = -\gamma h / \beta + (v - \delta) / \gamma + h \quad (24)$$

Proof. Differentiating (15) w.r.t. a gives, as a first order condition for an interior maximum:

$$L_a = (v - g - d)N_a + bQ_b + l [Q_b U(x) - Q_b U(y)] = 0 \quad (25)$$

From (7) we have:

$$N_a = 2h / (b + h - a)^2 \quad (26)$$

and:

$$N_b = -2h / (b + h - a)^2 \quad (27)$$

From (8) we have:

$$Q_a = Q_b = 1/2 \quad (28)$$

Noting that $x = y$ (from Proposition 2) and substituting these expressions into (25) yields:

$$\frac{(v - g - d)2h}{(b + h - a)^2} + b/2 = 0 \quad (29)$$

Differentiating (15) w.r.t. b gives, as a first order condition for an interior maximum:

$$L_b = -gV + (v - g - d)N_b + bQ_b + l [Q_b U(x) - Q_b U(y)] = 0 \quad (30)$$

Noting that $x = y$ (from Proposition 2), we may substitute from (7), (27) and (28) to obtain:

$$\frac{-g2h}{b + h - a} - \frac{(v - g - d)2h}{(b + h - a)^2} + b/2 = 0 \quad (31)$$

Adding (29) and (31) yields:

$$\frac{-g2h}{b + h - a} + b = 0 \quad (32)$$

$$\Rightarrow b + h - a = \frac{g^2 h}{b} \quad (33)$$

Substituting from (33) into (29) and re-arranging yields (23). Substituting (23) into (33) and re-arranging gives (24). Substituting (23) and (24) into (8) gives:

$$Q = (v - \delta)/\gamma + h \quad (34)$$

6.3 Competitive Equilibrium

To characterise a competitive equilibrium we take a multiplier μ for the expected profit constraint and form the Lagrangian:

$$M(p, \beta, a, b) = V(x, y) + \mu \Phi \quad (35)$$

Substituting for V and Φ yields:

$$\begin{aligned} M(p, \beta, a, b) = & Q(a, b)U(x) + (1 - Q(a, b))U(y) + \\ & \mu[p - \gamma bN(a, b) - \delta N(a, b) + v(N(a, b) - 1) - \beta(1 - Q(a, b))] \end{aligned} \quad (36)$$

It is now straightforward to establish:

Proposition 5. In competitive equilibrium the expected profit constraint is binding (i.e. $\Phi = 0$).

Proof. Differentiating (36) w.r.t. p yields one of the first order conditions for an interior maximum:

$$M_p = \frac{-QU'(x)}{D} - \frac{(1-Q)U'(y)}{D} + \mu = 0 \quad (37)$$

Since $U'(\cdot) > 0$, it follows that $\mu \neq 0$. Thus, by complementary slackness, it follows that the expected profit constraint is binding.

We now establish:

Proposition 6. In competitive equilibrium the warranty payment (β) is equal to $D(1 - \alpha)R$, and thus the consumer receives the same income stream regardless of whether the product breaks down or not, and is thus fully insured (i.e. $x = y$).

Proof. Differentiating (36) w.r.t. β yields another first order condition for an interior maximum:

$$M_b = \frac{(1-Q)U'(y)}{D} - m(1-Q) = 0 \quad (38)$$

Thus $\mu = U'(y)/D$. Substituting for μ in equation (37) gives $U'(x) = U'(y)$. Since $U''(.)$ is strictly negative, the function $U'(.)$ is invertible. Thus $x = y$ as required.

We now prove:

Proposition 7. Equilibrium production quality decisions and quality control decisions are the same under competitive equilibrium as under monopoly equilibrium.

Proof. Differentiating (36) w.r.t. a and b yields the remaining two first order conditions for an interior maximum:

$$M_a = m[-gN_a - dN_a + vN_a + bQ_a] = 0 \quad (39)$$

Noting that $m \neq 0$ and using (26), and (28) yields (29).

$$M_b = m[-g(N + bN_b) - dN_b + vN_b + bQ_b] = 0 \quad (40)$$

Noting that $m \neq 0$ and using (7), (27) and (28) yields (31).

Since equations (29) and (31) hold under competitive equilibrium, equations (23), (24) and (34) must also hold.

Equilibrium warranty payments, production quality decisions and quality control decisions are the same under competition as under monopoly, but the competitive price is lower: it falls until expected profits are zero.

6.4 Comparative Statics

Using Propositions 2 and 6 we may substitute for β in (23) and (24). This gives:

$$b = \gamma h / (D(1 - \alpha)R) + (v - \delta) / \gamma \quad (41)$$

$$a = -\gamma h / (D(1 - \alpha)R) + (v - \delta) / \gamma + h \quad (42)$$

From Proposition 3 it is clear that the demand curve facing the monopolist has the step function appearance of figure (6), and that the price p^* at which the step occurs is equal to DR . Note that the **market** demand curve facing a competitive industry has the same appearance, even though the competitive equilibrium price is lower than DR . Thus an increase in R is equivalent to an upward shift in the demand curve.

Inspection of (34), (35) and (36) yields the following comparative static results.

Proposition 5. (Supply side shifts)

- (a) If $v > \delta$ the sign of $\frac{db}{dg}$ is ambiguous but $\frac{da}{dg} < 0$, and $\frac{dQ}{dg} < 0$.
 (b) If $v < \delta$ then $\frac{db}{dg} > 0$ but the sign of $\frac{da}{dg}$ is ambiguous, and $\frac{dQ}{dg} > 0$.

For high rework values, an increase in production costs lowers the stringency of quality control and lowers the average quality of marketed products. For low rework values, an increase in production costs raises the production quality level and raises the average quality of marketed output.

- (c) $\frac{db}{dd} < 0$, $\frac{da}{dd} < 0$ and $\frac{dQ}{dd} < 0$.

An increase in quality control costs lowers the production quality level and the stringency of quality control, and lowers the average quality of marketed products.

- (d) $\frac{db}{dv} > 0$, $\frac{da}{dv} > 0$ and $\frac{dQ}{dv} > 0$.

An increase in rework value raises the production quality level and the stringency of quality control, and raises the average quality of marketed products.

Proposition 6 (Demand side shifts)

$$\frac{db}{dR} < 0, \frac{da}{dR} > 0 \text{ and } \frac{dQ}{dR} = 0.$$

An upward shift in the demand curve lowers the average production quality level but raises the stringency of quality control. These two effects exactly offset each other and the overall impact on the average quality of marketed products is zero.

7. CONCLUSIONS

It is frequently the case that product quality is unknown to consumers before they purchase the product. In this situation the determination of product quality and of warranty contracts need not be modelled as the consequence of exogenous type uncertainty. The determination of quality can be treated as an aspect of the firm's (expected) profit maximising decision by explicitly modelling the firm's quality management process. This process involves both production quality decisions and quality control decisions. Raising production quality and undertaking quality control are both costly, but they will still be undertaken because warranty commitments provide the appropriate incentive. Neither need there be legal compulsion to force firms to offer warranties. If buyers are more risk-averse than sellers, a demand for insurance arises which will induce sellers to offer warranties voluntarily.

An increase in production costs will lower the quality of marketed output for firms with high rework values and raise it for firms with low rework values. An increase in quality control costs or a decrease in rework value will lower production quality levels and induce less stringent quality control, resulting in a lower quality of marketed output. An upward shift in demand will lower production quality but raise the stringency of quality control. The two effects exactly cancel out and the quality of marketed output is unaffected.

The quality management process is complicated and the model presented here inevitably simplifies that process. Nonetheless quality management is central to the determination of the

quality of marketed output, and the economic analysis of product quality should therefore include a treatment of this aspect of the firm's behaviour.

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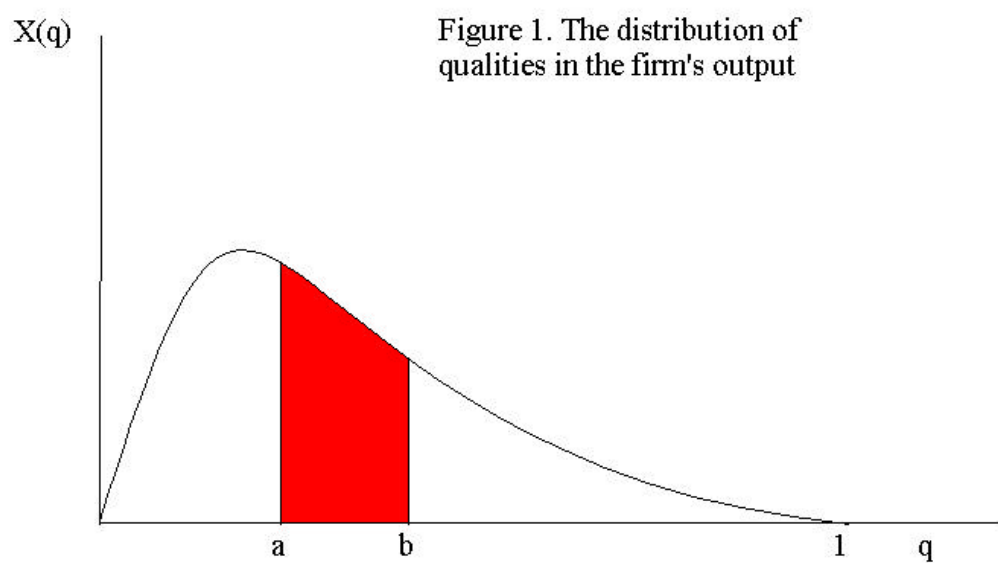


Figure 2. A "quality filter"

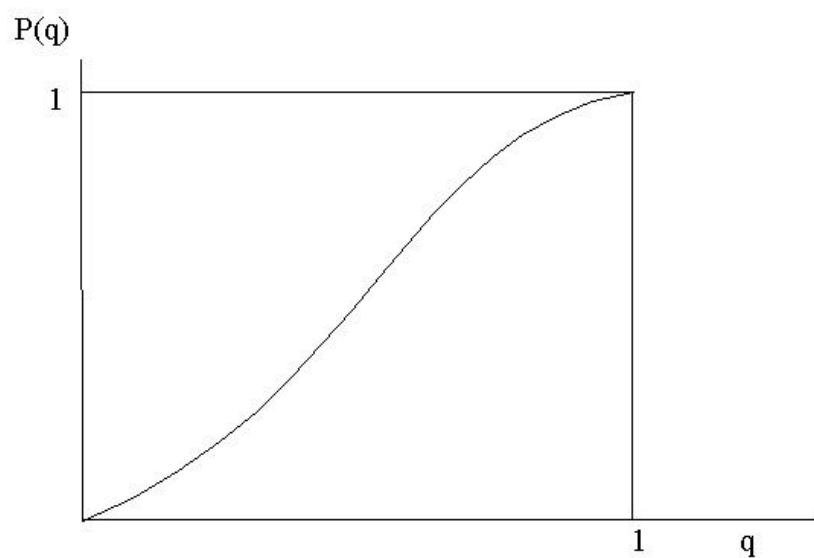


Figure 3.

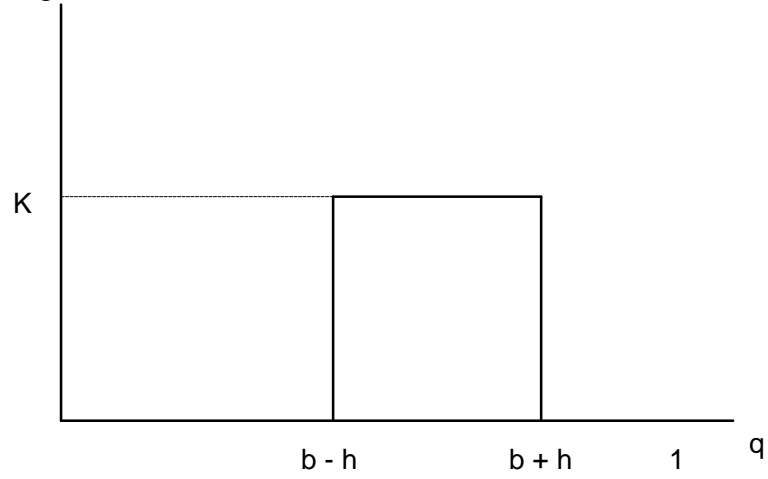


Figure 4.

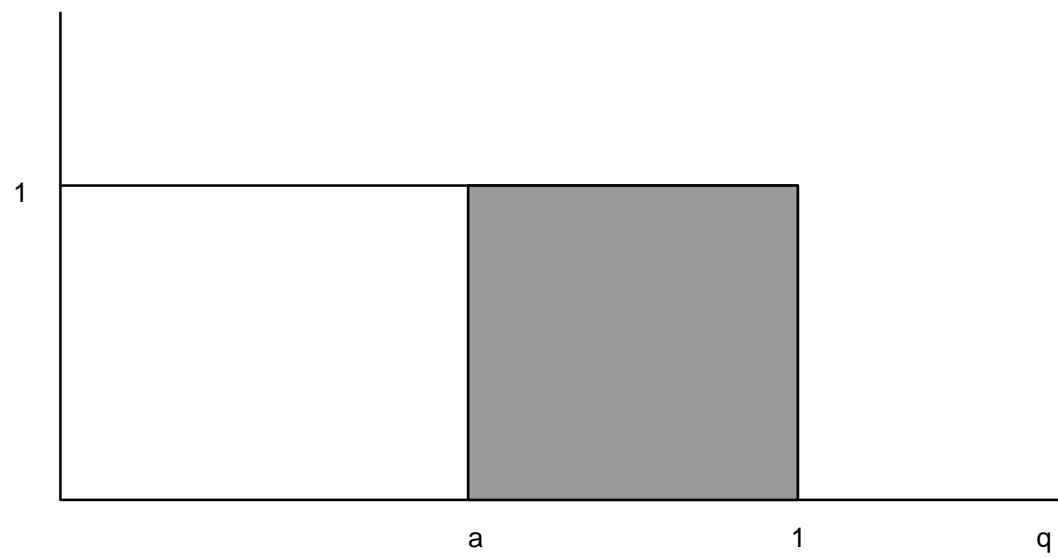


Figure 5.

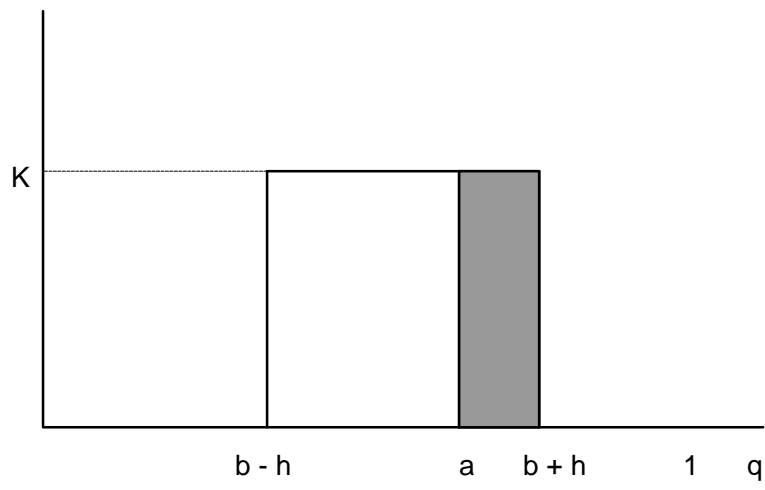


Figure 6.

